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A possible description of the quantum numbers in a hadronic string model

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ABSTRACT: We consider a critical composite superconformal string model to desribe hadronic interactions. We present a new approach of introducing hadronic quantum numbers in the scattering amplitudes. The physical states carry the quantum numbers and form a common system of eigenfunctions of the operators in this string model. We give explicit constructions of the quantum number operators.

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1 Introduction

In 1967 the Regge hadronic resonances have been discovered. The experimental data gave evidence of a typical relation between the spins $\mathcal{I}$ and the square of the masses M of these strongly interacting particles [1]. It turned out, that the resonances form the so-called Regge trajectories

$$\mathcal{I}(M^2) = \alpha_0 + \alpha' M^2 - n \ . \tag{1}$$

For $n = 0, 1, 2, ...$ one finds a family of parallel linear daughter trajectories. The constants α_0 and α' denote the intercept and the Regge slope. The leading ρ-meson trajectory corresponds to $n = 0$.

So far QCD has not been able to explain this phenomenon. On the other hand in the 1970's it was observed, that in string theory the mass shell condition

$$(L_0 - a)|\text{phys} >= 0$$

generates a mass spectrum with parallel trajectories. The intercept a depends on the cancellation condition of the conformal anomaly in the respective theory. Thus *hadronic strings* became a natural candidate for describing the Regge spectrum phenomena. However such efforts failed because of the necessary appearance of massless states of spin one and two, which do not correspond to physical hadronic resonances.

While these massless states gave rise to treat string theory as a fundamental theory of all interactions including gravity at Planck energy scale, the discrepance in hadronic string theory at typical strong interaction energy scales of $E \sim 1$ GeV remained unresolved for a long time. In [7], [10] one of the authors suggested a new critical composite superconformal hadronic string. It consists of the Neveu-Schwarz (NS) superstring and a fermionic superconformal string [5]. The NS field components are associated with the space-time degrees of freedom, while the fermionic sector carries the internal degrees of freedom, namely the hadronic quantum numbers. In accordance with the conformal anomaly cancelation in this model we have to impose new gauge constraints on the physical states. This allows to eliminate the problematic massless states from the string mass spectrum, which now becomes compatible with the Regge resonance spectrum [7].

In this paper we present a possible explicit construction of the operators, corresponding to the hadronic quantum numbers spin, isospin, electric

charge, hyper-charge, baryon charge, strangeness, charm, beauty and top. We give the structure of the hadronic wave functions.

In section 2 we review the main features of the composite string model and the structure of its superconformal generator. We discuss the cancellation of the anomaly achieved in $D = 10$ or $D = 4$ space-time dimensions and D' internal fermionic degrees of freedom. Then additional $\frac{D'}{6}$ or $\frac{D'}{6}-3$ conditions are needed respectively for the anomaly-free solution. In section 3 we give all gauge conditions eliminating ghosts in the physical states of the composite string model and the spectral equations for the quantum numbers.

2 The critical composite superconformal string model

In order to describe the physical hadronic states with their quantum numbers and masses we have to consider a string model in the four dimensional space-time. Such models can be obtained by compactification of the critical ten dimensional superstring. One way of compactification to four dimensions is the fermionization of six dimensions [2]-[3], namely, by introducing free world sheet fermions $\nu(\sigma,\tau)$ carrying all internal quantum numbers of the string [2], [5], [11]. A generalization and concrete realization of this approach has been achieved in the critical composite superconformal string model [10], [12]. This model unifies the superconformal structures of the Neveu-Schwarz operator $G_r^{(NS)}$ [9] and of the fermion operator $G_r^{(f)}$ [5] by introducing the composite superconformal operator

$$G_r = G_r^{(NS)} + G_r^{(f)}. \tag{2}$$

The operator $G_r^{(f)}$ is constructed in such a way, that G_r is a *singlet* in all quantum numbers, and

$$\{G_r^{(NS)}, G_r^{(f)}\} = 0.$$

The specific superconformal algebras of $G_r^{(NS)}$ and $G_r^{(f)}$ are closed separately (see the Appendix).

The canonical superconformal operator $G_r^{(NS)}$ reads [8],[9]

$$G_r^{(NS)} = \frac{1}{2\pi i}\oint H^\mu(\partial_\tau X_\mu)e^{+ir\tau}d\tau.$$

We use the notations

$$X_\mu^{(i)} = x_{0\mu}^{(i)} + ip_\mu^{(i)} \ln z_i + \sum_{n\neq 0} \frac{a_{n\mu}^{(i)}}{in} z_i^n, \;\; z_i = e^{-i\tau_i}$$

$$[a_{n\mu}^{(i)}, a_{m\nu}^{(i)}] = -n\, g_{\mu\nu}\delta_{n,-m}, \quad g_{00} = 1, \;\; g_{ii} = -1,$$

for the i-th string space-time coordinate of zero conformal weight, and

$$H_\mu^{(i)}(z_i) = b_{r\mu}^{(i)} z_i^r, \;\; \{b_{r\mu}^{(i)}, b_{s\nu}^{(i)}\} = -g_{\mu\nu}\delta_{r,-s}$$

for its superpartner in the Neveu-Schwarz string model. The NS-states are given by the Fock space of the products of the creation operators

$$\prod_{n,\mu}\prod_{m,\nu}\{a_{n\mu}^\dagger\}\{b_{m\nu}^\dagger\}|0> \;\; .$$

As usual the operator G_r of the composite string shall be of the conformal weight $I_c = 3/2$. Therefore the fermion operator $G_r^{(f)}$ is of the same conformal weight. It can be constructed as a three-linear combination [2] of the fermion fields ν_A of $I_c = 1/2$ and their currents J^{ν_A} of $I_c = 1$, i.e. [5]

$$G_r^{(f)}(\nu) = \sum_{A,B,C} \nu_A\nu_B\nu_C\varepsilon_{ABC} + \sum_A J^{\nu_A}\nu_A \;\; . \tag{3}$$

By construction $G_r^{(f)}$ is a singlet in the quantum numbers. The fermion operator generates all internal quantum numbers, what we show in detail in the next section.

We turn now to the description of the structure of the fields ν_A and their currents J^{ν_A}, entering in (3). Let $\psi_\alpha \equiv \psi_{\mu j}$, $\mu = 0,1,2,3$, $j = 1,2$ be Majorana spinor with $I_c = 1/2$. Its eight components are Lorentz spinors in μ and isospinors in j simultaneously. The respective currents J^{ν_A}

$$J^{\nu_A} = \tilde{\psi}_\alpha T_{\alpha\beta}\psi_\beta, \qquad \tilde{\psi}_\alpha = T_0\psi_\alpha = \gamma_0\tau_2\psi_\alpha,$$

are of conformal weight one. These currents are non-zero, if the respective 8×8 matrices $T_0 T_{\alpha\beta}$ are antisymmetric. The matrices $T_{\alpha\beta}$ can be choosen of the form

$$\gamma_\mu, \;\; \gamma_5\gamma_\mu\tau_i, \;\; \tau_i, \;\; \gamma_5\tau_i, \;\; [\gamma_\mu, \gamma_\nu].$$

Thus we obtain the 28 components for the vector, axial vector, scalar, pseudoscalar and tensor currents

$$J^{\nu_A} = \tilde{\psi}_\alpha T_{\alpha\beta} \psi_\beta = \{J^V_\mu, J^A_{\mu i}, J^S_i, J^P_i, J^T_{\mu\nu}\}.$$

They generate a Kac-Moody algebra, see [4]. With these currents we associate the respective 28 fermion components

$$\nu_A := \{\psi_\alpha; \phi_\mu, \rho_{\mu i}, \theta_i, \eta_i, \xi_{\mu\nu}\}\ , \quad \mu = 0,1,2,3, \quad i = 1,2,3. \tag{4}$$

These fields satisfy the standard anticommutation relations. We consider the physical quarks as compositions of the elementary field components (4).

We see, that the eight spinor components ψ_α generate 28 vector, axial vector, scalar, pseudoscalar and tensor fermionic components. Thus $G^{(f)}_r$ is a scalar in D'=36 fermionic components.

It is well known, that the Neveu-Schwarz string is critical in the space-time dimension $D = 10$. This comes from the condition of cancellation of the conformal anomaly, or equivalently the nilpotency of the BRST-charge $\Omega^2 = 0$ [3], [4]

$$\frac{3}{2}D - 26 + 11 = \frac{3}{2}(D - 10) = 0. \tag{5}$$

We point out, that the new composite string remains critical for D=10. This is due to the ghost contributions [3] from the introduced fermionic fields and their currents [10]. Indeed, in the composite model the condition (5) turns into

$$\frac{3}{2}D - 26 + 11 + (\frac{1}{2}D' - 3N) = \frac{3}{2}(D - 10) + (\frac{1}{2}D' - 3N) = 0, \tag{6}$$

where $-3N$ denotes the ghost contribution. In the dimension $D = 10$ the condition (6) is satisfied, if we choose $N = D'/6$ [10].

On the other hand, compactifying the string to the space-time dimension $D = 4$, we can impose $N = \frac{D'}{6} - 3$ gauge constraints. These gauge constraints on the fermionic sector are used to cancel the superfluous (in compairison

[3]The ghost contribution is given by $c_g = -2\epsilon(6I_c(I_c - 1) + 1)$, where $\epsilon = 1$ is the ghost statistic for integer I_c and $\epsilon = -1$ is the ghost statistic for half integer I_c. Hence N fermions with $I_c = 1/2$ contribute $c_g = -N$ and N corresponding currents with $I_c = 1$ contribute $c_g = -2N$.

with the Regge spectrum) states, and to shift the vacuum to $G_{-1/2}|0>$. This shift ensures the absence of a tachyon in the spectrum.

The amplitude of the hadron interactions are constructed by the operator formalism. The quantum numbers spin, isospin, electric charge, hypercharge, baryon charge, strangeness, charm, beauty and top and the masses of the hadronic states are included in the structure of the hadronic wave function which enters in the vertex operator structure, and hence in the structure of the N hadrons amplitude A_N.

The factorization of the amplitude gives the wave function of the corresponding hadronic state. This wave function contains a certain number of components, corresponding to its experimental mass. These components carry all hadron quantum numbers and will reproduce explicitly the quantum numbers in the hadronic amplitude structure. This has been shown in [6] where the superconformal string amplitudes for π mesons interactions have been constructed and in [11], [12] where the general N hadrons interaction amplitudes are obtained as multiparticle generalization of the Lovelace-Shapiro amplitude and are treated as composite superconformal strings.

Duality, crossing and cyclic symmetry for these superconformal composite string amplitudes hold together with the description of hadron quantum numbers.

3 The Quantum Numbers

In this section we give a possible explicit construction of the quantum numbers of hadron wave functions. This wave functions shall satisfy a set of gauge constraints

$$L_n|\text{phys}>= 0, \quad n > 0, \qquad G_r|\text{phys}>= 0, \quad r > 0, \tag{7}$$

which eliminate ghosts in the physical states. Here $L_n = L_n^{(NS)} + L_n^{(f)}$ denotes the Virasoro operator, the fermion part of which is explicitly given in the Appendix. As usual we impose the mass shell condition

$$(L_0 - 1/2)|\text{phys}>= 0, \tag{8}$$

where the intercept 1/2 is fixed by the nilpotency of the BRST charge.

In addition to this we have to require new gauge constraints

$$\nu_r^{(l)}|\text{phys}>=0, \qquad J^{\nu_r^{(l)}}|\text{phys}>=0, \;\; l=1,...,N=\frac{D'}{6}-3, \tag{9}$$

eliminating the ghosts contributions from the added fermionic fields and their currents. Such new constraints are in agreement with the anomaly cancellation condition (6). They are responsible for the reduction of the composite string spectrum to the Regge resonance spectrum. The gauge constraints (7)-(9) build a supermultiplet $L_n, G_r, J^{\nu_r^{(l)}}, \nu_r^{(l)}$ with the conformal weights $I_c = 2, \frac{3}{2}, 1, \frac{1}{2}$ respectively.

We search for a common system of eigenfunctions for (7)-(9) and the spectral equations

$$\hat{\mathcal{Q}}_m|\text{phys}>= q_m|\text{phys}> \tag{10}$$

on the remaining $D'-N$ fermion components. The operator $\hat{\mathcal{Q}}_m$ runs over the quantum number operators for the spin, isospin, electric charge, hypercharge, baryon charge, strangeness, charm, beauty and top.

An appropriate choice of the quantum number operators are those zero components of the corresponding currents

$$(\mathcal{J}^{\nu_A})_0 = T^{\nu_A} := \{G_r^{(f)}, (\nu_A)_{-r}, \} \tag{11}$$

which are Number operators, i.e. count the component carrying the spin, isospin, baryon charge and so on.

The operators (11) automatically commute with the Virasoro operators $L_n^{(f)}$ and $G_r^{(f)}$

$$[L_n^{(f)}, T^{\nu_A}] = 0, \quad [G_r^{(f)}, T^{\nu_A}] = 0,$$

what ensures the existence of a common system of eigenfunctions.

Since $G_r^{(f)}$ is a quantum number singlet, the charge T^{ν_A} carries the same quantum numbers as the field ν_A. Therefore the isospin operator shall be defined as

$$T_i^\eta := \{G_r^{(f)}, (\eta_i)_{-r}\}. \tag{12}$$

It generates the algebra $[T_i^\eta, T_j^\eta] = \varepsilon_{ijk}T_k^\eta$ and has eigenvalues

$$(\hat{T}_i^\eta)^2|\text{phys}>= T_i^\eta(T_i^\eta+1)|\text{phys}> .$$

Analogously the Lorentz spin operator is defined by

$$J \equiv (k_\mu T^{\xi^{\mu\nu}}) := \{G_r^{(f)}, (k_\mu \xi_{\mu\nu})_{-r}\} \tag{13}$$

with the momentum k_μ. The specific realization of the spin and isospin operators is given in the Appendix.

The baryon charge $B = 1$ is carried by hadron states with half-integer Lorentz spin. Among the fields in (4) only ψ is a Lorentz spinor. Therefore, if a wave function contains an even number of ψ components, it is a meson, otherwise it is a baryon. Thus the baryon charge is defined by

$$B = \frac{1}{2}(1 - (-1)^{N_B}) \tag{14}$$

where $N_B = \sum_l \tilde{\psi}_{-l}\psi_l$ is the number of spinor ψ-fields.

Among the currents (11) one can not find other operators with Number structure. On the other hand all 36 components (4) were used in the specific realization of (12)-(14). Therefore with them we can only construct meson and baryon states containing u and d quarks. To define the quantum numbers strangeness $\hat{s}$, charm $\hat{c}$, beauty $\hat{b}$ and top $\hat{t}$ independently, we have to extend the set (4) of 36 fermion components with an analogous partner-set of 36 field components (Lorentz scalars and isoscalars)

$$\omega_A := \{\chi_\alpha;\, \sigma_i, \sigma_{jk}\}\ , \quad \alpha = 1...8,\ i, j, k = 1...7. \tag{15}$$

The fermion fields (15) show the following anticommutation properties

$$\{(\tilde{\chi}_\alpha)_r, (\chi_\beta)_s\} = \delta_{\alpha\beta}\delta_{r,-s}, \qquad \tilde{\chi}_\alpha = \chi_\alpha\gamma_0\tau_2,$$

$$\{(\sigma_i)_r, (\sigma_j)_s\} = \delta_{ij}\delta_{r,-s},$$

$$\{(\sigma_{ij})_r, (\sigma_{kl})_s\} = (\delta_{ik}\delta_{jl} - \delta_{il}\delta_{jk})\delta_{r,-s}, \quad \sigma_{ij} = -\sigma_{ji}.$$

From the 8 component spinor χ_α we can define 28 components of the Kac-Moody currents

$$J^{\omega_A} = \Big\{J_i := \tilde{\chi}\Gamma_i\chi; \quad J_{jk} := \tilde{\chi}[\Gamma_i, \Gamma_j]\chi\Big\}. \tag{16}$$

which are Lorentz scalars and isoscalars. Here Γ_i denote the 8×8 Clifford matrices, obeying the standard anticommutation relations $\{\Gamma_j, \Gamma_k\} = 2\delta_{jk}$.

The currents J_i, J_{jk} carry the same quantum numbers as the fields σ_i, σ_{jk}. This guaranties the scalarity of the superconformal operator

$$G_r^{(f)}(\omega) = \sum_{ABC} \omega_A\omega_B\omega_C\epsilon_{ABC} + \sum_A J^{\omega_A}\omega_A. \tag{17}$$

Composing the superconformal operators (3) and (17)

$$G_r^{(f)} = G_r^{(f)}(\nu) + G_r^{(f)}(\omega) \tag{18}$$

we describe the full set of hadron quantum numbers [4].

From (12) the third isospin component in the ν_A-space is given by

$$T_3^\eta := \{G_r^{(f)}, (\eta_3)_{-r}\}.$$

By construction $\sigma_{12} \in \omega_A$ is the partner-component to η_3. Therefore we define the isospin projection in ω_A by

$$T_{12} := \{G_r^{(f)}, (\sigma_{12})_{-r}\}. \tag{19}$$

The full projection of the isospin of the hadron state corresponds to the sum

$$\hat{T}_3|\text{phys}>= (T_3^\eta + T_{12})|\text{phys}> \ . \tag{20}$$

In order to describe the flavour quantum numbers strangeness, charm, beauty and top we have to construct four commutating operators in ω_A with structure similar to the zero component of the currents J^{ω_A}. We will find them using the properties of the spinor representation of the $O(6)$ group. The 8 component spinors in the compactified $D = 6$ space transforms under $O(6)$. The generators of this group are given via the 8×8 Γ_i matrices by $[\Gamma_i, \Gamma_j] = 4iM_{ij},\ \ i, j = 1...6$. It holds $[M_{ij}, M_{kl}] = 0$, if and only if $i \neq k, l$ and $j \neq k, l$. Thus among the generators one can choose not more then three commutating with each other M_{ij}, e.g. M_{12}, M_{34}, M_{56}. Since the Clifford matrices Γ_i transforms under $O(6)$ as vectors

$$[M_{ij}, \Gamma_k] = i(\delta_{ik}\Gamma_j - \delta_{jk}\Gamma_i), \quad i, j, k = 1...6,$$

neither of the matrices $\Gamma_1, ..., \Gamma_6$ commutes with all of the three generators M_{12}, M_{34}, M_{56}. On the other hand, the matrix $\Gamma_7 := \prod_{i=1}^6 \Gamma_i$ commutes with all generators M_{ij}. Thus Γ_7 is the fourth independent operator.

[4] $G_r^{(f)}$ is a quantum number singlet in $D' = 72$, which can be reduced to D=10 according to (6) by imposing $N = D'/6 = 12$ conditions on the physical states, i.e. 6 on each of the sets (4) and (15). Then for the compactification of the composite string to D=4 are used $N = D'/6 - 3 = 9$ constraints.

According to this we introduce first three independent generators T_{12}, T_{34}, T_{56}. We choose T_{12} according to (19), since it defines the s, c, b, t-flavour part of the isospin projection. The remaining two generators are given by

$$T_{34} := \{G_r^{(f)}, (\sigma_{34})_{-r}\} \tag{21}$$
$$T_{56} := \{G_r^{(f)}, (\sigma_{56})_{-r}\}. \tag{22}$$

The fourth charge operator has the form

$$T_7 := \{G_r^{(f)}, (\sigma_7)_{-r}\}. \tag{23}$$

It commutes with (19), (21), (22) and with L_n, G_r. Hence there exists a common system of eigenfunctions for all these operators.

The operators $T_{12}, T_{34}, T_{56}, T_7$ can not directly be identified as flavor quantum number operators. In fact one has to consider the following four linear combinations, which define strangeness, charm, beauty and top

$$\hat{s} := -\frac{1}{2}(T_{34} + T_{56}) - \frac{1}{2}(T_{12} + T_7), \tag{24}$$

$$\hat{c} := -\frac{1}{2}(T_{34} + T_{56}) + \frac{1}{2}(T_{12} + T_7), \tag{25}$$

$$\hat{b} := -\frac{1}{2}(T_{12} + T_{34}) + \frac{1}{2}(T_{56} + T_7), \tag{26}$$

$$\hat{t} := +\frac{1}{2}(T_{12} + T_{56}) - \frac{1}{2}(T_{34} + T_7). \tag{27}$$

Thus the physical quarks are described as superpositions of elementary string fields. Hence the quarks are composite objects in this model.

If we substitute (20) and (24)-(27) into the expression for the electric charge

$$Q = T_3 + (\hat{s} + \hat{b} - \hat{c} - \hat{t} + B)/2$$

with the baryon charge B, we obtain

$$Q = T_3^\eta + B/2. \tag{28}$$

This can be taken as a definition, which is independent of the quark flavors contained in the hadronic state. Analogously the expression for the hypercharge in the quark model

$$Y = \hat{s} + \hat{b} - \hat{c} - \hat{t} + B$$

together with (24)-(27) gives

$$Y = B - 2T_{12}. \tag{29}$$

In case of $T_{12} = 0$ we have $Y = B$ for all states, containing only u- and d-quarks.

The above constructions give a realization of the hadron quantum numbers $Q, B, T_3, \hat{s}, \hat{b}, \hat{c}, \hat{t}$ in terms of the composite string operators. Each given hadron state can be interpreted as a wave function with the respective quantum numbers in the composite string model.

The spin, isospin and baryon charge in the first generation of $\left(\begin{smallmatrix} u \\ d \end{smallmatrix}\right)$-quarks are realized from the ν_A-set, while the s, c, b, t flavours are realized on the partner set ω_A. Thus the respective wave functions have the structure

$$< \text{meson}| =< 0, k|F_{\nu_A} F_{\omega_A}, \quad < \text{baryon}| =< 0, k|\psi F_{\nu_A} F_{\omega_A}$$

with $< 0, k| =< 0| \exp(ikx)$. In this F_{ν_A} is a compositions of components from (4), symmetric or antisymmetric with respect to the spin and isospin, while F_{ω_A} carries the quantum numbers $\hat{s}, \hat{c}, \hat{b}, \hat{t}$, and the isospin projection T_{12}, and contains only components from ω_A.

The fermion superconformal operator $G_r^{(f)}$ has $D' = 72$ internal degrees of freedom. Notice, that the quark model contains 6 flavors, 3 colours and 4 spin components, what also gives total number of 72 degrees of freedom.

Finally we turn to the realization of the mass-spectrum of the hadron resonances. Let the strange fermion component gives a contribution of $\Delta M^2 = 0.3$ GeV^2 to the mass of the physical state. This agrees with the experimental fact of parallel ρ-meson- and K^*-trajectories

$$M_{K^*}^2 - M_\rho^2 = \frac{1}{2\alpha'} = 0.3\text{GeV}^2.$$

In this case the composite string model gives a much better description of the physical mass spectrum then the quark model. In the quark model these trajectories are not parallel.

4 Discussion

A description of the hadron quantum numbers has been given in the composite string model in the simplest case of 8 component spinors in a compactified six-dimensional space. Another possibility is to consider 32 component

spinors, what is natural in the critical D=10 string. There the number of currents obtained as pairs of 32 component spinors is equal to 496. This gives $D' = 528$ fermionic degrees of freedom. Here we impose $N = D'/6 = 88$ gauge conditions on physical states. This allows a simultaneous description of the quarks and leptons in terms of the fermionic sector of the model. This is a subject of further publications.

5 Appendix

We give here the concrete realization of the described operators. In [5] the fermion superconformal operator $G_r^{(f)}(\nu)$ was introduced as

$$G_r^{(f)}(\nu) = \frac{1}{\sqrt{7}}\{\frac{1}{4i}(\tilde{\psi}\hat{\phi}\psi) + \frac{1}{4}(\tilde{\psi}\gamma_5\tau_i\hat{\rho}\psi) + \frac{1}{4}(\tilde{\psi}\tau_i\psi)\eta_i + \frac{1}{4}(\tilde{\psi}\gamma_5\tau_i\psi)\theta_i +$$

$$+\frac{1}{8i}(\xi^{\mu\nu}\tilde{\psi}\frac{1}{2}[\gamma_\mu,\gamma_\nu]\psi) + \frac{1}{i}(\phi\rho_i)\theta_i + \frac{1}{2i}(\theta_i\theta_j\eta_k)\varepsilon_{ijk} - \frac{1}{2i}((\rho_i\rho_j)\eta_k)\varepsilon_{ijk} +$$

$$+\frac{1}{6i}(\eta_i\eta_j\eta_k)\varepsilon_{ijk} - \frac{1}{2i}(\xi^{\mu\nu}\rho_{\mu i}\rho_{\nu i}) - \frac{1}{2i}(\xi^{\mu\nu}\phi_\mu\phi_\nu) + \frac{1}{6i}(\xi^{\mu\nu}\xi_{\nu\lambda}\xi_\mu^\lambda)\}_r$$

with

$$(\eta_i\eta_j\eta_k)_r = \sum_{r_1+r_2+r_3=r}(\eta_i)_{r_1}(\eta_j)_{r_2}(\eta_k)_{r_3}.$$

By analogy the operator $G_r^{(f)}(\omega)$ is given by

$$G_r^{(f)}(\omega) = \frac{1}{\sqrt{7}}\{(\tilde{\chi}\Gamma_i\chi\sigma_i)_r + (\tilde{\chi}[\Gamma_j,\Gamma_k]\chi\sigma_{jk})_r + (\sigma_i\sigma_j\sigma_{ij})_r + (\sigma_{ij}\sigma_{jk}\sigma_{ki})_r\},$$

where the matrices Γ_i are defined as

$$\Gamma_i = (\Gamma_{i'},\Gamma_\mu),\ i' = 1,2,3,\quad \mu = 4,5,6,7,$$

$$\Gamma_{i'} = \gamma_5\otimes\tau_{i'},\quad i' = 1,2,3,$$

$$\Gamma_4 = \gamma_1\otimes 1,\ \Gamma_5 = \gamma_2\otimes 1,\ \Gamma_6 = \gamma_3\otimes 1,\ \Gamma_7 = \Gamma_1\Gamma_2\Gamma_3\Gamma_4\Gamma_5\Gamma_6.$$

The full fermion operator is given by (18). Its superconformal algebra is closed

$$\{G_r^{(f)}, G_s^{(f)}\} = 2L_{r+s}^{(f)} + \frac{D'}{6}(r^2 - \frac{1}{4})\delta_{r,-s},$$

i.e. the superconformal structure of the model is conserved.

The Virasoro conformal algebra is satisfied

$$[L_n^{(f)}, L_m^{(f)}] = (n-m)L_{n+m}^{(f)} + \frac{D'}{24}(n^3-n)\delta_{n,-m},$$

and the conformal and superconformal operators commutation relation hold

$$[L_n^{(f)}, G_r^{(f)}] = (\frac{n}{2} - r)G_{n+r}^{(f)}.$$

The factor $\frac{n}{2} - r$ implies, that G_r is of the conformal weight 3/2.

The Virasoro operator $L_n^{(f)}$ correspond to the fermion part of the action of the free string

$$S = \frac{i}{8\pi\alpha'}\int d\tau d\sigma(\tilde{\psi}\hat{\partial}\psi - \phi\hat{\partial}\phi - \rho\hat{\partial}\rho - \eta\hat{\partial}\eta + \theta\hat{\partial}\theta + \xi\hat{\partial}\xi + \chi\hat{\partial}\chi + \sigma_i\hat{\partial}\sigma_i + \sigma_{ij}\hat{\partial}\sigma_{ij}).$$

Next we give the realization of the the quantum number operators in terms of the components of the elementary string fields. The isospin operator T_i^η is given by

$$T_i^\eta = \sum_{j,k} :(\eta_j\eta_k\varepsilon_{ijk})_0: + \frac{1}{2}\sum_{l>0}(\tilde{\psi}_{-l}\tau_i\psi_l) + \sum_{j,k} :(\theta_j\theta_k + \rho_{\mu j}\rho_{\mu k})_0\varepsilon_{ijk}: .$$

The Lorentz-spin operator is defined as

$$(k_\mu T_{\mu\nu}^\xi) = -\frac{1}{4}\sum_{l>0} k_\mu\tilde{\psi}_{-l}[\gamma_\mu, \gamma_\nu]\psi_l + \sum :(k_\mu\rho_{\mu i}\rho_{\nu i} + k_\mu\phi_\mu\phi_\nu + k_\mu\xi_{\mu\lambda}\xi_{\lambda\nu})_0: .$$

The operators $T_{12}, T_{34}, T_{56}, T_7$ can be realized as

$$T_{12} = \sum_{r>0}\tilde{\chi}_{-r}\tau_3\chi_r + \sum :(\sigma_1\sigma_2)_0: + \sum :(\sigma_{1k}\sigma_{2k})_0:,$$

$$T_{34} = \sum_{r>0}\tilde{\chi}_{-r}[\Gamma_3, \Gamma_4]\chi_r + \sum :(\sigma_3\sigma_4)_0: + \sum_k :(\sigma_{3k}\sigma_{4k})_0:,$$

$$T_{56} = \sum_{r>0}\tilde{\chi}_{-r}[\Gamma_5, \Gamma_6]\chi_r + \sum :(\sigma_5\sigma_6)_0: + \sum_k :(\sigma_{5k}\sigma_{6k})_0:,$$

$$T_7 = \sum_{r>0}\tilde{\chi}_{-r}\Gamma_7\chi_r + \sum_k :(\sigma_{7k}\sigma_k)_0: .$$

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